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Research article

MODELLING PERMEABILITY AND DISPERSION OF POTASSIUM AND PATHOGEN DEPOSITION IN WARRI, NIGER DELTA OF NIGERIA

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Abstract

Pathogen deposition in soil and water has lots of different roles in the environments, but most microbes are hazardous to human, the focus of the study is on the hazardous dimension of microbes, the formation in Warri Nigeria are prone to high degree of permeability due to the homogenous nature of the formation, the study location experience lots of environmental factors at negative dimension that affect humans in the depositions of phreatic formation, aquiferious zone become harmful to human due to high rates of contaminants in the study area, the formation established several variations base on high degree of deltaic nature of the formations, To solve this challenges, mathematical model were found suitable in the study area, this concept were applied through the developed governing equations, several conditions that influence the system were considered in the developed governing equation, the express equation were derived in accordance with several conditions considered to influence the behaviour of pathogens and potassium in the study location, the study is imperative because the derived mathematical model will streamline various challenges of microbial migrations and deposition influence of micronutrients in the formations. The expressed model will be useful to assess the condition of micronutrient and its influence on pathogenic growth in the study area. **Copyright © IJSEE, all right reserved.**

Keywords: modelling permeability, dispersion potassium and pathogen deposition

1. Introduction

Microorganisms have dominated the biosphere for 3.5 billion years. In view of these 3.5 billion years of microbial evolution, it seems that there are more different types of microorganisms than anyone ever imagined (Hackl et al. 2004; Schloss and Handelsman 2004; Spear et al. 2003; Ward 2002; Ward et al. 1998 Vijay,2007). In addition, the earth's habitats present complex gradients of environmental conditions that include extreme variations in temperature, light, pH, pressure, salinity, and both inorganic and organic compounds. Evolutionary mechanisms have made it possible for microorganisms to survive these extreme variations by means of genetic adaptation (Bond et al. 2000; Brofft et al. 2002; Hallberg and Johnson 2001; Ward et al. 1998). Microorganisms play very important roles in environmental conservation and protection. Interest in predicting the fate and transport of bacteria in the subsurface area is motivated by either a concern that microbes can contaminate drinking water supplies or their role in bioremediation (Fontes et al. 1991). Bacterial transport is affected by the propensity for cell sorption within the pore environment in the subsurface (Grasso and Smets 1998; Grasso et al. 1996; Karickhoff et al. 1979; Smets et al. 1999 Vijay, 2007). Bacterial strains with different cell surface properties show different adhesion kinetics and affinity for substrate (Chen and Strevett 2001). Bacterial surface physicochemical properties can be chemically modified to stimulate or impede bacterial adhesion to the substratum (Powelson and Mills 1998; van der Mei et al. 2001; Whitekettle 1991).

2. Theoretical Background

Micronutrients deposition in soil and water environment serves in different varieties under the influence of permeability coefficient and dispersion rate in soil and water , micronutrients of this types were found to deposit predominantly under the influence of high degree of permeability coefficient in the formation producing high percentage of dispersions in the study location, this are generated from natural origin, but increase in pollutant rate of micronutrients increase the deposition of pathogen concentration this was confirmed from risk assessment carried out to monitor the growth rate of some microbes rapidly increasing in microbial population in the study area., this condition implies that the deposition of pathogen predominantly deposited in the study location, but the focus of this study centre on the health implication of pathogen in soil at optimum level in saturated zone to aquiferous environment. Predominant deposition of potassium from man made activities and natural origin has rapidly increase contaminant from micronutrients to the optimum level in the study area, this condition has worsened the contamination of water quality in the study area. To stop this plague, mathematical model were developed to mathematically develop a model that monitor the deposition of pathogen and influence of potassium in phreatic aquifers. The model was developed base on the parameters that influences on the migration of pathogen and deposition of potassium in phreatic aquifers. The microbial growth rate were expressed from the developed equation that generated the developed model, since the micronutrients rapidly increase the concentration of pathogen in soil and water environment, the model express the rate deposition at various formations under various influence that sustain the deposition of micronutrients and microbes in the formations. The model were discretize in phase base on various influence including the behaviour of the microbes in their transport process. This will definitely stream line the Behaviour of microbes in terms of producing conceptual frame work that can be applied in ensuring ground water monitoring in the study location, in other to solve contaminant problem in ground water abstraction used for human activities.

Nomenclature

θb	=	Soil bulk density unitless
$V\theta$	=	Overall volumetric mass coefficient transfer
V	=	Velocity
K	=	Permeability
D	=	Dispersion
C	=	Concentration of salmonellae

$Cw\theta$ = Potassium concentration in liquid phase

T = Time

Z = Distance

3. Governing equation

$$\theta b\frac{\partial C}{\partial t} = V\theta\frac{\partial C}{\partial z} + V\frac{\partial C}{\partial z} - K\frac{\partial C}{\partial z} + D\frac{\partial C}{\partial t} + Cw\theta\frac{\partial C}{\partial z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\theta b\frac{\partial C_1}{\partial t} = V\theta\frac{\partial C_1}{\partial z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\left.\begin{aligned} t &= 0 \\ z &= 0 \\ C_{(o)} &= 0 \\ \frac{\partial C_1}{\partial t}\Big| t &= 0,\ B \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\theta b\frac{\partial C_2}{\partial t} = V\frac{\partial C_2}{\partial z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$\left.\begin{aligned} t &= 0 \\ z &= 0 \\ C_{(o)} &= 0 \\ \frac{\partial C_2}{\partial t}\Big| t &= 0,\ B \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$\theta b\frac{\partial C_3}{\partial t} = -K\frac{\partial C_3}{\partial z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$\begin{aligned} t &= 0 \\ z &= 0 \\ Cs_{(o)} &= 0 \\ \frac{\partial C_3}{\partial t}\Big| t &= 0,\ B \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$\theta b\frac{\partial C_4}{\partial t} = Cw\theta\frac{\partial C_4}{\partial z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$\left.\begin{aligned} t &= 0 \\ z &= 0 \\ C_{(o)} &= 0 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

The application of this techniques is to split equations according to various condition that influence the microbes in various stratification of the formation in phreatic aquifers, this condition were found necessary since the substrate has an interaction with the contaminant growth in the formations, subject to this relation, the rate of concentration are found to reflect on the growth rate of microbes in soil and water environments, so it is imperative to ensure that the substrate is thoroughly examined to monitor the rate of deposition at various formation, thus predict their depositions at different depths in the study area.

$$\frac{\partial C_4}{\partial t}\Big| t = 0, B$$

$$D\frac{\partial C_5}{\partial t} + Cw\theta\frac{\partial C_5}{\partial z} \qquad \text{.................. (10)}$$

$$\left.\begin{aligned} t &= 0 \\ z &= 0 \\ C_{(o)} &= 0 \\ \frac{\partial C_5}{\partial t}\Big| t &= 0, B \end{aligned}\right\} \qquad \text{.................. (11)}$$

$$D\frac{\partial C_6}{\partial t} - K\frac{\partial C_6}{\partial z} \qquad \text{.................. (12)}$$

$$\left.\begin{aligned} t &= 0 \\ z &= 0 \\ C_{(o)} &= 0 \\ \frac{\partial C_6}{\partial t}\Big| t &= 0, B \end{aligned}\right\} \qquad \text{.................. (13)}$$

$$D\frac{\partial C_7}{\partial t} + V\theta\frac{\partial C_7}{\partial z} \qquad \text{.................. (14)}$$

$$\left.\begin{aligned} t &= 0 \\ z &= 0 \\ C_{(o)} &= 0 \\ \frac{\partial C_7}{\partial t}\Big| t &= 0, B \end{aligned}\right\} \qquad \text{.................. (15)}$$

$$D\frac{\partial C_8}{\partial t} + V\frac{\partial C_8}{\partial z} \qquad \text{.................. (16)}$$

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$$\left.\begin{array}{l} t = 0 \\ z = 0 \\ C_{(o)} = 0 \\ \frac{\partial C_8}{\partial t}\Big| t = 0,\ B \end{array}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

Applying direct integration on (2) we have

$$\theta b \frac{\partial C}{\partial t} = V\theta + K_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

Again, integrate equation (18) directly yield

$$\theta bC = V\theta + K_1 + K_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

Subject to equation (3), we have

$$C_o = K_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

And subjecting equation (19) to (3)

At $\left.\frac{\partial C_1}{\partial t}\right| \begin{array}{l} = 0\ C_{(o)} = C_o \\ t = 0 \end{array}$

Yield

$$0 = VCs_o + K_2$$

$$\Rightarrow K_2 = -VC_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

So that we put (20) and (21) into (19), we have

$$C_1 = \theta bC_1 t - V\theta t + C_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

$$C_1 - \theta b = C_o - V\theta t \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

$$\Rightarrow C_1\left[C_1 - \theta bt\right] = C_o\left[C_1 - V\theta\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

$$\Rightarrow Ct = C_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

$$\theta b \frac{\partial Cs_2}{\partial t} = V\frac{\partial C_2}{\partial z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

We approach the system using the Bernoulli's method of separation of variables.

i.e. $C_2 = ZT$ ……………… (26)

$$\frac{\partial C_2}{\partial t} = ZT^1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

$$\frac{\partial C_2}{\partial z} = Z^1 T \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

Put (27) and (28) into (26), so that we have

$$\theta b\, ZT^{1} = VZ^{I}T \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

$$\theta b \frac{T^{1}}{T} = V \frac{Z^{1}}{Z} = -\lambda^{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (30)$$

Hence

$$\theta b \frac{T^{1}}{T} = -\lambda^{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

$$VZ^{1} + \lambda^{2} Z = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

From (32)

$$T = ACos \frac{\lambda t}{\theta b} + B\ Sin \frac{\lambda z}{\theta b} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (33)$$

And (32) gives

$$T = C\ell \frac{-\lambda^{2}}{\theta b} t \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (34)$$

By substituting (32) and (33) into (26)

$$C_{2} = \left[ACos \frac{\lambda}{\sqrt{\theta b}} t + B\,Sin \frac{\lambda}{\sqrt{\theta b}} z \right] C_{o} \ell^{\frac{-\lambda^{2}}{\sqrt{\theta b}} t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \ (35)$$

$$C_{o} = Ac \qquad \ldots\ldots\ldots\ldots\ldots\ldots \ (36)$$

Equation (35) becomes

The expression in Equation (2) applied direct integration, this is to ensure that parameters involved express there functions in accordance with the system, direct integration were found necessary to combine the variables, such similarities were observed in there various functions, this to ensure the parameters achieve there results under the influence of micronutrient and microbial behaviour in those formations, the deposition of the substrate reflect on the concentration of the microbes from soil were the concentration of potassium and pathogen deposit, this two parameters experience high degree of concentration. Variable that were found to express their relation with each other it develop pressure of increase in deposition of potassium and microbial population base on the geologic influence in the study area..

$$C_{2} = C_{o} \ell \frac{-\lambda^{2}}{V} t \ Cos \frac{\lambda}{V} z \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (37)$$

Again at $\left. \frac{\partial C_{2}}{\partial t} \right| \begin{matrix} = 0,\ z = 0 \\ t = 0,\ B \end{matrix}$

Equation (37) becomes

$$\frac{\partial C_{2}}{\partial t} = \frac{\lambda}{\theta b} C_{o} \ell^{\frac{-\lambda^{2}}{V} t} \ Sin \frac{\lambda}{\theta b} z \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (38)$$

i.e. $0 = \frac{\lambda}{\sqrt{\theta b}} Sin \frac{\lambda}{\theta b} 0$ ………………… (39)

$C_o \frac{\lambda}{\sqrt{\theta b}} \neq 0$ Considering NKP

$0 = -C_o \frac{\lambda}{\theta b} Sin \frac{\lambda}{\theta b} B$ ………….. (40)

$\lambda = \frac{n\pi\sqrt{\theta b}}{2}$ ………………… (41)

So that equation (38) becomes

$C_2 = C_o \ell^{\frac{-n^2\pi^2\theta b}{2V}} Cos \frac{n\pi\sqrt{V}}{2\sqrt{\theta b}} z$ ………………… (42)

$C_2 = C_o \ell^{\frac{-n^2\pi^2\theta b}{2V}} Cos \frac{n\pi}{2} z$ ………………… (43)

$\theta b \frac{\partial C_3}{\partial t} = K \frac{\partial C_3}{\partial z}$ ………………… (6)

We approach the system by using Bernoulli's method of separation of variables.

$C_3 = ZT$ ………………… (44)

$\frac{\partial C_3}{\partial t} = ZT^1$ ………………… (45)

$\frac{\partial C_3}{\partial z} = Z^1 T$ ………………… (46)

Hence, we put (45) and (46) into (44), so that we have

$\theta b \frac{ZT^1}{T} = K \frac{Z^1 T}{T}$ ………………… (47)

i.e. $\theta b \frac{T^1}{T} = K \frac{Z^1}{Z} - \lambda^2$ ………………… (48)

Hence $\theta b \frac{T^1}{T} + \lambda^2 = 0$ ………………… (49)

i.e. $Z + \dfrac{\lambda^2}{\theta b} Z = 0$ ………………… (50)

And $\theta b T^1 + \lambda^2 T = 0$ ………………… (51)

From (50) $Z = ACos \dfrac{\lambda}{\theta b} Z + BSin \dfrac{\lambda}{\theta b} Z$ ………………… (52)

And (45) gives

$$T = C_o \ell^{\frac{-\lambda^2}{K} t} \quad \text{………………… (53)}$$

By substituting (52) and (53) into (44), we get

$$C_3 = \left[ACos \frac{\lambda}{\theta b} Z + B\,Sin \frac{\lambda}{\sqrt{\theta b}} Z \right] Cos \ell^{\frac{-\lambda^2}{K} t} \quad \text{………………… (54)}$$

Subject (54) to condition in (6) so that we have

$$C_o = Ac \quad \text{………………… (55)}$$

Similar conditions are expressed in equation (55) the depositions of potassium transporting to unconfined bed are found to deposit very high concentration of substrate, due to high degree of permeability deposition, therefore the tendency of high concentration from pathogens and potassium can be attributed to high degree of saturation that will always allow fast migration of the contaminants to pheratic formations, similar condition developed the composition of these parameter integration in equation (55) express the concentration of pathogen at initial concentration, this are determined by the rate of stratification porosity of the formation. , so the formation stratum determined the expressed variable that developed model denoted as C_s = Ac in equation (55).

Equation (55) becomes

$$C_3 = C_o \ell \frac{-\lambda^2}{K} t \; Cos \frac{\lambda}{\theta b} Z \quad \text{………………… (56)}$$

Again at $\dfrac{\partial C_3}{\partial t} \Big| \; t = 0, B$

Equation (56) becomes

$$\frac{\partial C_2}{\partial t} = \frac{\lambda}{\sqrt{\theta b}} Cos \ell^{\frac{-\lambda^2}{K} t} \; Sin \frac{\lambda}{\theta b} x \quad \text{………………… (57)}$$

i.e. $0 = -C_o \frac{\lambda}{\sqrt{\theta b}} Sin \frac{\lambda}{\theta b} 0$

$C_o \frac{\lambda}{\sqrt{\theta b}} \neq 0$ Considering NKP

Equation (40) and (57) express the influence of the substrate in terms of increase in microbial population, this condition were considered in various conditions in the formations of the soil, microbial population developed increase from the substrate influence, such situation generated microbial predominant. The equations take care of the rate of potassium deposition in the formations, equation (40) and (55) expressed the results of high degree of deposition in the formations, the above expressed equation reflect the consequences of potassium deposition, expressing the tendency of increasing of microbial population, this are determined by high rate of microbes feeding from the substrate deposition in the formations. This condition generates lots of variations in microbial behaviour in different dimensions. Moreso the degree of substrate considered in the state of microbial transport determined the rate of concentration at various strata in soil and water environment.

Which is the substrate utilization for microbial growth rate (population) so that

$$0 = -C_o \frac{\lambda}{\theta b} Sin \frac{\lambda}{\theta b} B \quad \text{.............. (58)}$$

$$\Rightarrow \frac{\lambda}{\sqrt{\theta b}} = \frac{n\pi}{2} \quad \text{.............. (59)}$$

$$\Rightarrow \frac{\lambda}{\sqrt{\theta b}} = \frac{n\pi\sqrt{\theta b}}{2} \quad \text{.............. (60)}$$

So that equation (61)

$$C_3 = C_o \ell^{\frac{-n^2\pi^2\theta b}{2K}} Cos \frac{n\pi\sqrt{\theta b}}{2\sqrt{\theta b}} Z \quad \text{.............. (61)}$$

$$\Rightarrow C_3 = C_o \ell^{\frac{-n^2\pi^2\theta b}{2K}} t \, Cos \frac{n\pi}{2} Z \quad \text{.............. (62)}$$

Now we consider equation (8)

$$\theta b \frac{\partial C}{\partial t} = Cw\theta \frac{\partial C}{\partial z} \quad \text{.................... (8)}$$

Using Bernoulli's method of separation of variables, we have

$$C_4 = ZT \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (63)$$

$$\frac{\partial C_4}{\partial t} = ZT^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (64)$$

$$\frac{\partial C_4}{\partial Z} = Z^1 T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (65)$$

$$\theta bZT = -Cw\,\theta Z^1 T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (66)$$

i.e. $$\theta b\frac{T^1}{T} = Cw\,\theta\frac{Z^1}{Z} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (67)$$

$$\theta b\frac{T^1}{T} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (68)$$

$$Cw\,\theta\frac{Z^1}{Z} = \varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (69)$$

And $$Z = B\ell^{\frac{\varphi}{Cw\theta}Z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (70)$$

Put (68) and (69) into (63), gives

$$C_4 = A\ell^{\frac{\varphi}{Cw\theta}Z} \quad B\ell^{\frac{\varphi}{Cw\theta}t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (71)$$

$$C_4 = AB\ell^{(z-t)}\frac{\varphi}{Cw\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (72)$$

Subject equation (69) to (8) yield

$$C_4 = (o) = C_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (73)$$

So that equation (73) becomes

$$C_4 = C_o\ell^{(t-z)}\frac{V}{Cw\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (74)$$

Now, we consider equation (10)

$$D\frac{\partial C}{\partial t} + Cw\theta\frac{\partial C}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots \quad (10)$$

Apply Bernoulli's method, we have

$$C_5 = ZT \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (75)$$

$$\frac{\partial C}{\partial t} = ZT^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (76)$$

$$\frac{\partial C}{\partial Z} = Z^1 T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (77)$$

Put (75) and (76) into (10), so that we have

$$DZT^{1} = -Z^{1}T\,Cw\theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (78)$$

i.e. $$D\frac{T^{1}}{T} = \frac{Z^{1}}{Z}Cw\theta = \phi \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (79)$$

$$D\frac{T^{1}}{T} = \phi \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (80)$$

$$Cw\theta\frac{Z^{1}}{Z} = \phi \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (81)$$

$$T = \frac{\phi}{D}t \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (82)$$

And $$Z = B\ell\frac{-\phi}{Cw\theta}Z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (83)$$

Put (80) and (81) into (73), gives

$$C_{5} = A\frac{\phi}{Cw\theta}t \;\; B\frac{-\phi}{Cw\theta}t \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (84)$$

$$C_{5} = AB\ell^{(z-t)}\frac{\phi}{Cw\theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (85)$$

Subject equation (83) and (84) into (74) yield

$$C_{5} = (o) = C_{o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (86)$$

So that equation (84) and (85) becomes

$$C_{5} = (o) = C_{o}\;\ell^{(z-t)}\frac{\phi}{Cw\theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (87)$$

Now, we consider equation (12)

$$D\frac{\partial C_{6}}{\partial t} - K\frac{\partial C_{6}}{\partial z} \qquad \ldots\ldots\ldots\ldots \quad (12)$$

Applying Bernoulli's method of separation of variables, we have

$$C_{6} = ZT \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (88)$$

$$\frac{\partial C_{6}}{\partial t} = ZT^{1} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (89)$$

$$\frac{\partial C_{6}}{\partial Z} = Z^{1}T \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (90)$$

$$DZT^{1} - KZ^{1}T \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (91)$$

i.e. $D\frac{T^1}{T} = K\frac{Z^1}{Z}$ .... (92)

$$D\frac{T^1}{T} = \alpha \quad \text{.................} \quad (93)$$

$$K\frac{Z^1}{Z} = \alpha \quad \text{.................} \quad (94)$$

And $Z = B\ell^{\frac{\alpha}{D}Z}$ .... (95)

Put (94) and (95) into (88) gives

$$C_6 = A\ell^{\frac{\alpha}{K}t*} \; B\ell^{\frac{\alpha}{K}t} \quad \text{.................} \quad (96)$$

$$C_6 = AB\ell^{(z-t)}\frac{\alpha}{K} \quad \text{.................} \quad (97)$$

Subject equation (95) and (96) into (97) yield

$$C_6 = (o) = C_o \quad \text{.................} \quad (98)$$

So that equation (95 and (98) becomes

$$C_6 = C_o\,\ell^{(t-z)\frac{\alpha}{K}} \quad \text{.................} \quad (99)$$

We consider equation (14)

$$D\frac{\partial C}{\partial t} + V\theta\frac{\partial C}{\partial z} = 0 \quad \text{...........} \quad (14)$$

$$C_7 = ZT \quad \text{.................} \quad (100)$$

$$\frac{\partial C_7}{\partial t} = ZT^1 \quad \text{.................} \quad (101)$$

$$\frac{\partial C_7}{\partial Z} = Z^1T \quad \text{.................} \quad (102)$$

Put (100) and (101) into (14), so that we have

$$DZT^1 = V\theta Z^1T \quad \text{.................} \quad (103)$$

i.e. $D\frac{T^1}{T} = V\theta\frac{Z^1}{Z}$ .... (104)

$$D\frac{T^1}{T} = \rho \quad \text{.................} \quad (105)$$

$$V\theta \frac{Z^1}{T} = \rho \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (106)$$

$$T = A\frac{\rho}{D}t \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (107)$$

And $Z = B\ell^{\frac{-\rho}{V\theta}z}$ ……………… (108)

Put (106) and (107) into (100), gives

$$C_7 = A\ell^{\frac{\rho}{V\theta}t} \; B\ell^{\frac{\rho}{V\theta}z} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (109)$$

$$C_7 = AB\ell^{-(z-t)}\frac{\rho}{V\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (110)$$

Subject equation (107) and (109) into (100) yield

$$C_7 = (o) = C_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (111)$$

So that equation (109) and (110) becomes

$$C_7 = A\ell^{\frac{\rho}{V\theta}t} \; B\ell^{\frac{\rho}{V\theta}z} \quad \ldots\ldots\ldots \quad (112)$$

Now, we consider equation (16) which is the steady plow rate of the system

$$D\frac{\partial C_8}{\partial t} + V\frac{\partial C_8}{\partial z} \quad \ldots\ldots\ldots \quad (16)$$

Applying Bernoulli’s method, we have

$$C_8 = ZT \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (113)$$

$$\frac{\partial C_8}{\partial t} = ZT^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (114)$$

$$\frac{\partial C_8}{\partial Z} = Z^1T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (115)$$

Put (113) and (114) into (16), so that we have

$$DZT^1 = V\,Z^1T \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (116)$$

i.e. $D\frac{T^1}{T} = V\frac{Z^1}{Z}$ ……………… (117)

$$D\frac{T^1}{T} = \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (118)$$

$$V\frac{Z^1}{Z} = \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (119)$$

$$Z = A\frac{\theta}{D}Z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (120)$$

And $T = B\frac{\theta}{V}t \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (121)$

Put (119) and (121) into (113), gives

$$C_8 = A\ell^{\frac{\theta}{D}} \bullet B\ell^{\frac{\theta}{V}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (122)$$

$$C_8 = AB\ell^{(t-z)}\frac{\theta}{V} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (123)$$

Subject to equation (122) and (123) yield

$$C_8 = (o) = C_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (124)$$

So that equation (123) become

$$C_8 = C_o\ell^{(t-z)}\frac{\theta}{V} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (125)$$

Expressions in (125) show case of the Steady state considered in the equation as expressed in the system. The depositions of substrate were expressed under the influences of formation variation in deposition in the strata. But in most situation the formation may experienced homogeneous deposition including potassium maintained homogeneity concentration in some formation, it implies that in phreatic aquifers the tendency of uniform flow of the substrate and microbial concentration in the formation are most likely to be experiences , therefore such condition may result to uniform flow and concentration between the substrate and pathogen concentration, so equation (125) expressed such condition in the system, this reflect the behaviour assumed in the migration of the contaminants and the deposition of potassium in the study location.

Now, assuming that at the steady flow, there is no NKP for substrate utilization, our concentration is zero, so that equation (124) becomes

$$C_8 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (126)$$

The expression in equation (126) were able to consider the situation substrate deposition were not experienced, the microbes may experiences adaptation, but if it cannot adapt, it will either migrate to the next formation or degrade in population, this condition are possible in the sense that some formations the substrate may experienced inhibition, thus the concentration will become zero, it implies that there is no deposition of substrate in those formation as expressed in equation (126)

Therefore, solution of the system is of the form

$$C = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7 + C_8 \qquad \ldots\ldots\ldots\ldots\ldots \quad (127)$$

We now substitute (25), (43), (62), (74), (87), (99), (112) and (125) into (128), so that we have the model of the form

$$C = C_o + C_o \ell \frac{-n^2\pi^2\theta b}{2V} Cos \frac{n\pi}{2} Z + C_o \ell \frac{-n^2\pi^2\theta b}{2K} Cos \frac{n\pi}{2} Z$$

$$+ C_o \ell^{(t-z)} \frac{\theta b}{Cw\theta} + C_o \ell^{(z-t)} \frac{\phi}{Cw\theta} + C_o \ell^{(t-z)} \frac{\alpha}{K} +$$

$$C_o \ell^{(t-z)} \frac{\rho}{V\theta} + C_o \ell^{(t-z)} \frac{\theta}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (128)$$

$$\Rightarrow C = C_o \; 1 + \ell \frac{-n^2\pi^2\theta b}{2V} t + Cos \frac{n\pi}{2} Z + C_o \; \ell \frac{-n^2\pi^2\theta b}{2K} t \; Cos \frac{n\pi}{2} Z +$$

$$C_o \ell^{(t-z)} \frac{\theta b}{Cw\theta} + C_o \ell^{(z-t)} \frac{\phi}{Cw\theta} + C_o \ell^{(t-z)} \frac{\alpha}{K} + C_o \ell^{(t-z)} \frac{\rho}{V\theta} + C_o \ell^{(t-z)} \frac{\theta}{V} \quad \ldots\ldots \quad (129)$$

The derived model in (129) is from the customized equation that considered a number of conditions that could influence the deposition of potassium in the study location. The deposition of potassium were examined thoroughly from different conditions in the study location, these process were itemizes, thus modifying the developed governing equation, numerous conditions that influence the behaviour of potassium deposition were also expressed in the system, since potassium are substrate to microbial growth thus determined the population of the microbe in soil and water environments, these condition were streamlined in the derived model at various stage, the behaviour of pathogen deposition express the concentration variables denoted mathematically in the system, this condition were determined through the boundary values as express in the model equation, different phase were expressed on the process of developing the model denoting it through various mathematical tools, from various characteristics of the formations, the rate of concentration of the potassium determined the rate of concentration of the microbes under normal condition, situations were the deposition are very high and there is degradation of the microbes this were also considered in the system as it was expressed on the derived mathematical expression. The model if applied will definitely monitored the migration of pathogen and determine growth rate of pathogen in phreatic aquifers.

4. Conclusion

Potassium expressions influencing the deposition of pathogen are usually deposited in our environment through made activities and natural origin. Potassium depositions are usually from made activities, the rates of concentration in the study area were confirmed to be very high, and the investigation from risk assessment generated the rate of potassium concentration. The level of deposition are reflected on high deposition of pathogen in the study area, the migration of pathogen in soil and water varies, these expression were examined from risk assessment, these conditions were reflected from poor quality of water in the study location, the ugly scourge were noticed from ground water analyzsis from different locations, results proof that the quality of water are very poor, the contaminant investigated are microbial deposition of pathogen predominant in the study location, this condition implies that high deposition of potassium increase the concentration of the microbes from organic soil to phreatic aquifers in the study area, the derived mathematical models developed will definitely solve the challenges of pollution and determine the concentration of potassium in phreatic aquifers.

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